Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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TOTAL	

General Certificate of Education
Advanced Level Examination
January 2011

Mathematics MFP4

Unit Further Pure 4

Friday 28 January 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions in the spaces provided.

1 Let $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix}$.

(a) Use a row operation to show that $(x+y+z)$ is a factor of Δ. *(2 marks)*

(b) Hence, or otherwise, express Δ as a product of linear factors. *(2 marks)*

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QUESTION PART REFERENCE

2 The non-zero vectors $\mathbf{a}$ and $\mathbf{b}$ have magnitudes a and b respectively.

Let $c = |\mathbf{a} \times \mathbf{b}|$ and $d = |\mathbf{a} \, . \, \mathbf{b}|$.

By considering the definitions of the vector and scalar products, or otherwise, show that

$$c^2 + d^2 = a^2 b^2$$

(3 marks)

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3 (a) Find the values of t for which the system of equations

$$\begin{aligned} tx + 2y + \quad 3z &= a \\ 2x + 3y - \quad tz &= b \\ 3x + 5y + (t+1)z &= c \end{aligned}$$

does not have a unique solution. *(3 marks)*

(b) For the integer value of t found in part **(a)**, find the relationship between a, b and c such that this system of equations is consistent. *(3 marks)*

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4 The non-singular matrix $\mathbf{X} = \begin{bmatrix} 3 & 1 & -3 \\ 2 & 4 & 3 \\ -4 & 2 & -1 \end{bmatrix}$.

(a) (i) Show that $\mathbf{X}^2 - \mathbf{X} = k\mathbf{I}$ for some integer k. *(3 marks)*

(ii) Hence show that $\mathbf{X}^{-1} = \frac{1}{20}(\mathbf{X} - \mathbf{I})$. *(2 marks)*

(b) The 3×3 matrix $\mathbf{Y}$ has inverse $\mathbf{Y}^{-1} = \begin{bmatrix} 60 & 0 & 0 \\ 0 & 0 & -10 \\ 0 & 20 & 0 \end{bmatrix}$.

Without finding $\mathbf{Y}$, determine the matrix $(\mathbf{XY})^{-1}$. *(3 marks)*

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5 The planes Π_1 and Π_2 have vector equations $\mathbf{r} \cdot \begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix} = 5$ and $\mathbf{r} \cdot \begin{bmatrix} 10 \\ -1 \\ -11 \end{bmatrix} = 4$ respectively.

(a) Write down cartesian equations for Π_1 and Π_2. *(1 mark)*

(b) Find a vector equation for the line of intersection of Π_1 and Π_2. *(5 marks)*

(c) The plane Π_3 has cartesian equation $5x + 3y + 11z = 28$.

Use your answer to part **(b)** to find the coordinates of the point of intersection of Π_1, Π_2 and Π_3. *(4 marks)*

(d) Determine a vector equation for the plane which passes through the point (4, 1, 9) and which is perpendicular to both Π_1 and Π_2. *(3 marks)*

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6 The plane Π has equation $\mathbf{r} \,.\, \begin{bmatrix} 12 \\ 15 \\ 16 \end{bmatrix} = 11$ and the point Q has coordinates $(1,\ 1,\ -1)$.

(a) Show that Q is in Π. *(1 mark)*

(b) (i) Write down cartesian equations for the line l which passes through Q and is perpendicular to Π. *(2 marks)*

(ii) Deduce the direction cosines of l. *(2 marks)*

(c) The points M and N are on l, and each is 50 units from Π.

Find the coordinates of M and N. *(3 marks)*

(d) Given that the point $P(5,\ 1,\ -4)$ is in Π, determine the area of triangle PMN. *(3 marks)*

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7 Let $\mathbf{Y} = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$.

(a) Show that 4 is a repeated eigenvalue of **Y**, and find the other eigenvalue of **Y**. *(7 marks)*

(b) For each eigenvalue of **Y**, find a full set of eigenvectors. *(5 marks)*

(c) The matrix **Y** represents the transformation T.

Describe the geometrical significance of the eigenvectors of **Y** in relation to T. *(3 marks)*

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8 The plane transformation T is represented by the matrix $\mathbf{M} = \begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix}$.

(a) The quadrilateral $ABCD$ has image $A'B'C'D'$ under T.

Evaluate $\det \mathbf{M}$ and describe the geometrical significance of both its sign and its magnitude in relation to $ABCD$ and $A'B'C'D'$. *(3 marks)*

(b) The line $y = px$ is a line of invariant points of T, and the line $y = qx$ is an invariant line of T.

Show that $p = \frac{1}{2}$ and determine the value of q. *(5 marks)*

(c) (i) Find the 2×2 matrix $\mathbf{R}$ which represents a reflection in the line $y = \frac{1}{2}x$. *(2 marks)*

(ii) Given that T is the composition of a shear, with matrix $\mathbf{S}$, followed by a reflection in the line $y = \frac{1}{2}x$, determine the matrix $\mathbf{S}$ and describe the shear as fully as possible. *(5 marks)*

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END OF QUESTIONS